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C-curve and independent oil-rate and oil-cut decline

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Abstract

Decline analysis is an important working tool, particularly for reserves evaluation in producing fields. However, its theoretical foundations are not fully understood and much work remains to be done to develop the theory and application of the method.

Based on practical experience of fitting decline curves to both field production and reservoir simulation results, the authors have identified two short-comings of conventional decline analysis, together with partial solutions to these problems. It is estimated that the two improvements together will lead to a significant improvement in the accuracy of decline analysis reserves evaluation.

The first problem is that of the limitations of hyperbolic curves and other alternatives to exponential decline. When field production or reservoir simulator results show non-exponential decline, the decline rate usually reduces in time (e.g. from 25% per annum to 15%, 10% etc), until levelling out and then continuing at a more-or-less constant rate (e.g. 5% per annum). In contrast, hyperbolic curves show decline rates that keep on reducing, eventually heading to a zero decline rate. In consequence, hyperbolic decline curves can give rise to unrealistically high ultimate recovery values. Instead of hyperbolic curves, an alternative generalisation of exponential decline is presented – the “C” or “cumulative” curve. These curves take a simple form; they include hyperbolic curves as a subset; and they allow the late-time decline rates to approach a non-zero asymptote.

The second problem is that there are cases where fitting a decline curve to the oil-cut curve gives a very unrealistic prediction of oil-rate, and vice-versa. A way of addressing the problem that has been found to be very successful is to fit independent decline curves to oil-cut and oil-rate and then use

both simultaneously to predict how the gross-liquid production rate varies with time.

Introduction

Although there are many extensions of decline analysis, incorporating pressures, material balance etc as discussed in Mittar and Anderson⁴, most practical applications of decline analysis use only a single equation to calculate production rate or oil-cut from cumulative production (or some mathematically equivalent calculation). The two main applications are

- To calculate ultimate recovery and the expected production profiles for producing wells and fields whose production history shows that decline has begun.
- For new fields and wells, to calculate the approximate expected production profile over the whole production lifetime. For this application, which is very important in exploration, data room work and multi-field studies, it is necessary to make an external estimate of ultimate recovery.

There are not that many forms of the basic decline equation in use. The simple exponential equation is probably the most widely used, especially for calculations with new fields and wells.

When decline is clearly non-exponential, the Arps¹ curves – hyperbolic and harmonic – are still by far the most widely used equations, 65 years after they were presented as an empirical extension of exponential decline. More recently, Li and Horne³ and Fekane and Tiab² have derived alternative equations.

We will focus our attention on the exponential, hyperbolic and Li-Horne equations. The harmonic equation is a special case of hyperbolic (the limit as the Arps b parameter goes to 1) and, moreover, has the unsatisfactory behaviour that production goes to infinity as time goes to infinity. The Fekane-Tiab equation is interesting, but too complicated for our purposes. Unlike the other equations, it cannot be easily manipulated to give e.g. cumulative oil as a function of time. The problem is that the equation linking production rates to time cannot be analytically integrated.

The exponential, hyperbolic and Li-Horne equations take the following forms for expressing oil production rate, $q(t)$, as a function of cumulative production.

Exponential decline –

$$q(t) = \alpha \cdot (UR - Np) \quad (1)$$

Hyperbolic decline

$$q(t) = \alpha \cdot \sqrt[1-b]{(UR - Np)} \quad (2)$$

Li-Horne decline

$$q(t) = \frac{\alpha}{Np} - \beta \quad (3)$$

The shapes of the different curves are illustrated in Fig.1 below.

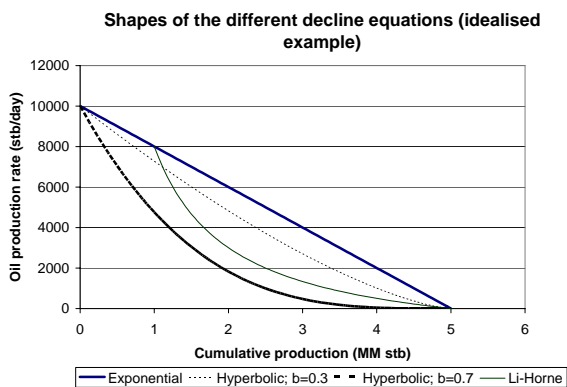


Figure 1

Limitations of hyperbolic and other existing decline curves

Although all that we ultimately need from the equations is to give the ability to match the decline history of real wells and fields, it is unfortunately the case that all of the curves suffer shortcomings.

Although it is difficult to get real production data out to very high water-cuts (above 99.9%) and low oil-rates (because wells and fields are abandoned), it is easy to create the next best thing – a simulator profile run out to such water-cuts and oil-rates. We can then rely on the fact that the simulator should capture, at least approximately, all the physics of reservoir performance and can expect the late production life seen in simulator models to be similar to what would be seen in reality.

A typical decline plot (oil rate vs cumulative oil) from a simulator model (a sector model of the Ebughu North-East field in offshore Nigeria) is given in Fig. 2.

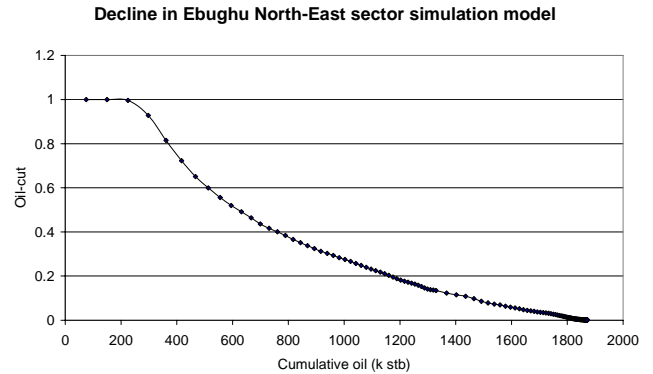


Figure 2

Exponential decline, which is limited to being a straight-line on the oil-cut vs cumulative plot, would, if fitted, for example, to production data between $Np = 300,000$ stb and $Np = 500,000$ stb would seriously underestimate ultimate recovery.

Li-Horne decline, while it allows for some curvature in the decline plot, has, in its current form, only two adjustable parameters. Hence, if the rate and the ultimate recovery are chosen, the degree of curvature of the curve is fixed. Moreover, the Li-Horne curve suffers the following disadvantages

- It cannot approximate exponential decline. One is forced into a discontinuous choice “Is this well suitable for exponential decline or for Li-Horne decline.
- Early in production life, when Np is low, the Li-Horne equation calculates enormously high production rates. Hence, it cannot be used to generate approximate production profiles for new wells, unless further assumptions are made.

Hyperbolic decline allows a range of curvatures in the decline plot, depending on the value of b . However, it suffers the weakness that the hyperbolic equation implies that the gradient of production rate vs cumulative production goes to zero as Np approaches UR . Note that this gradient is identical to the conventional decline rate, since

$$\frac{dq}{dNp} = \frac{dq}{dt} \cdot \frac{dt}{dNp} = \frac{dq}{dt} \cdot \frac{1}{q} \quad (4)$$

The C-curve formulation

Normally, there is an easy way round this problem, namely to use a high “ UR ” value. With fields that have highly non-exponential decline, this can give rise to the situation where, if you add up all the well decline “ UR ” values, the result is in excess of the STOIP. An example of such a field is the Alba field in the North Sea, which has oil of 6 cp insitu viscosity and underlying water.

Robertson, Cunningham et al⁵ describe how the development planning of the Alba Extreme South extension gave the impetus for the development of C-curves. In order to examine

the impact of a full range of development uncertainties and options, an approximate “system” model of the field and production facilities. Such a model needs to be able, when calibrated to reservoir simulation results, to generate approximate production profiles for a range of development options (chiefly, variations in the number of wells and facility capacities) and a range of STOIPs etc.

The approach taken was to make an estimate of the field ultimate recovery as a function of the number of wells drilled, calculate an expected UR per new well and then use decline curves to generate production profiles. Similar approaches are used with great success for fields showing exponential decline, especially for clusters of gas-fields.

However, the fact that hyperbolic equations gave rise to artificially high UR values prevented this approach from working. Hence, the C-curve equation (for “cumulative” curve) was derived, as a simple, analytically integrable equation that contains the exponential and hyperbolic equations as special cases, but is not obliged to show gradient of production rate vs cumulative production going to zero as N_p approaches UR. The equation takes the form

$$q(t) = \alpha.(UR - Np) + \beta.(UR - Np)^{b+1} \quad (5)$$

Its derivation is given in Appendix A.

Figure 3 below illustrates how C-curves give a better match than hyperbolic curves to reservoir simulation profiles, when UR is limited to physically reasonable values. (The plot is the reversal of the plots above, showing $r = (UR-Np)/UR$, with dr/dx equivalent to oil-cut.

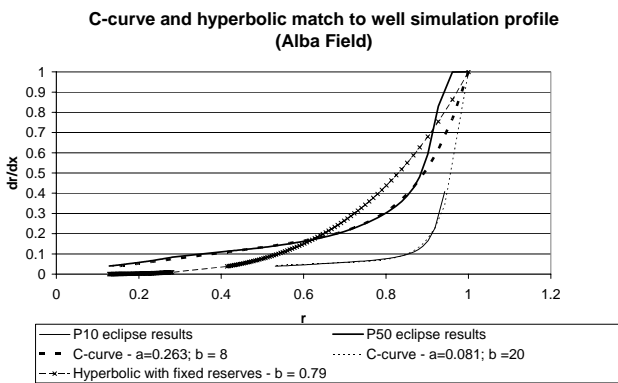


Figure 3

Conflicting oil-cut and oil-rate declines

Another problem that arises is when a well shows conflicting oil-cut and oil-rate declines. An example is given below. Fig 4 shows a reasonable match for the oil-rate decline for the well EB-05HT in the Addax-operated Ebughu field, but it can be seen, in Fig 5, that, with a single decline curve, the corresponding oil-cut decline is very unrealistic.

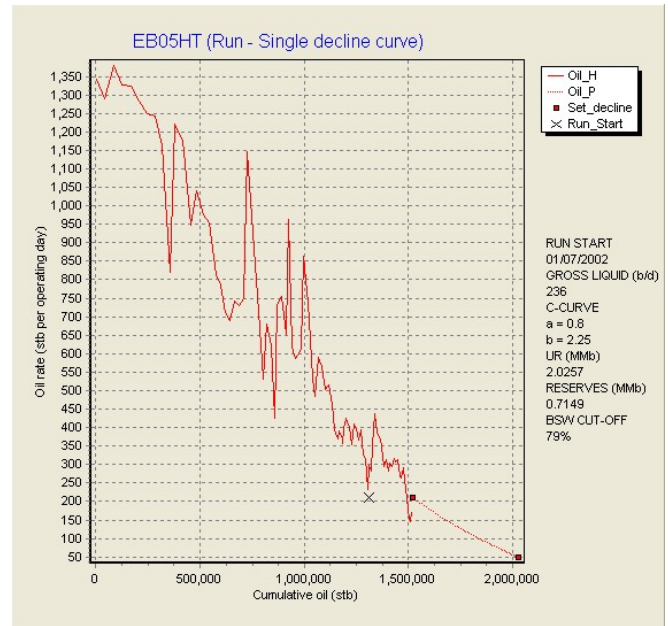


Figure 4

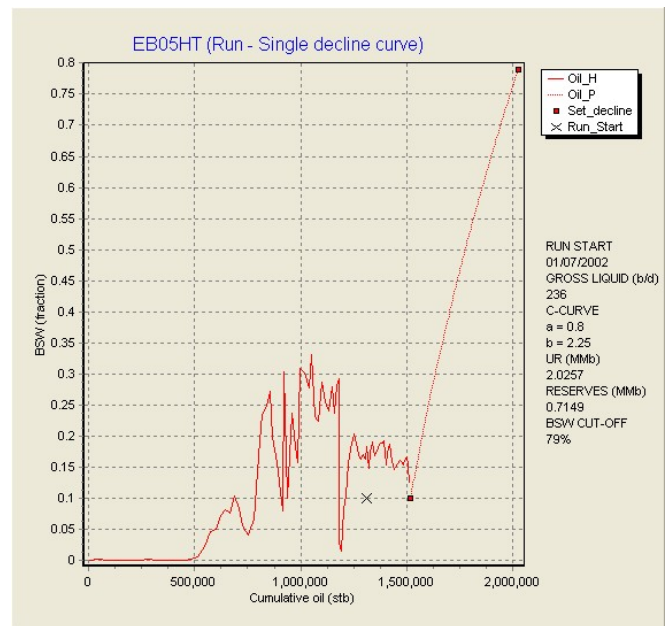


Figure 5

The result of the bad fit to the oil-cut decline is an unrealistic prediction of water production, as illustrated in the Fig. 6 below. While water production forecasts are less important than oil production forecasts, they may still have a considerable impact, particularly if there are facilities constraints, as there are with the Ebughu field.

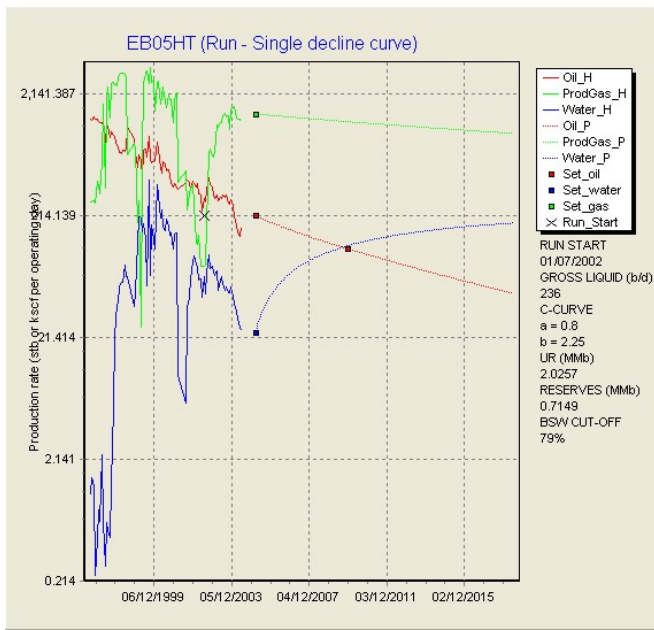


Figure 6

Independent models of oil-cut and oil-rate decline

A simple way round this problem is to fit separate decline curves to oil-cut and oil-rate. The two decline curves can then be combined to yield a prediction of gross liquid rate (GLR) and water rate, as follows and as illustrated in Figures 7 and 8 below.

1. A trend is fitted to the BSW versus Cumulative Oil and the Oil Rate versus Cumulative Oil production data.
2. From a Cumulative Oil value a BSW and an Oil Rate values are obtained using the trend curves.
3. Using those two values the water rate is calculated:

$$BSW = \text{Water Rate} / (\text{Water Rate} + \text{Oil Rate})$$

$$\Leftrightarrow \text{Water Rate} = BSW \times \text{Oil Rate} / (1 - BSW)$$
4. The Oil Rate and Water Rate versus Time curves are built.

With this method the GLR is allowed to vary with time, which makes it more flexible to use.

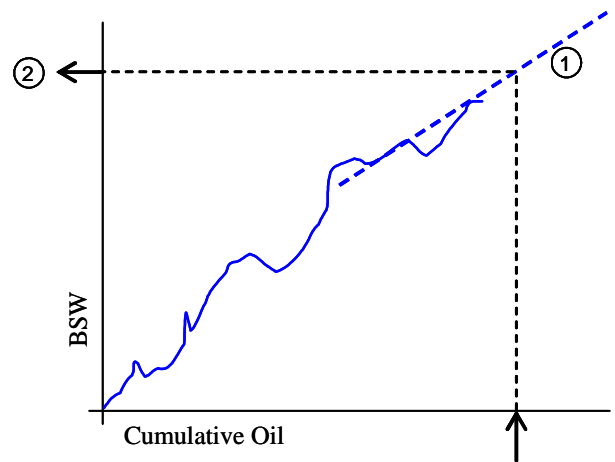


Figure 7

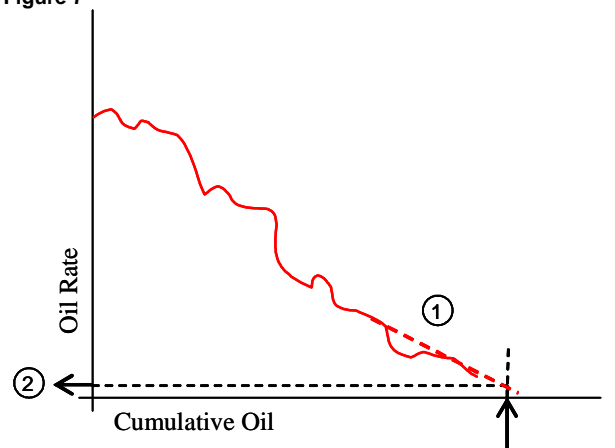


Figure 8

The following field example is the same as in the previous section, this time using the independent oil cut and oil rate decline method. The trend fit to the oil-rate versus cumulative oil is the same as in the previous case. This time a separate curve is also fitted to the oil rate versus cumulative oil data. The combination of both results in a plot of rates versus time (Fig. 9) that shows a much more reasonable prediction for water production.

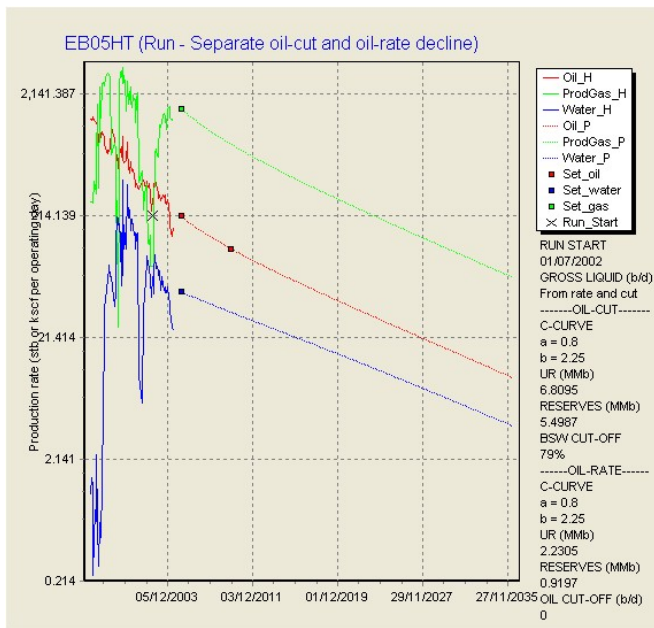


Figure 9

While the method is generally robust and is part of the standard method of decline analysis used by Addax Petroleum (using the Serafim “FUTURE” production forecasting application), it should be borne in mind that it is possible to specify decline curves that give rise to very large gross-liquid production rates late in production life, as illustrated in Fig. 10 below. This situation arises whenever oil-cut reserves are lower than oil-rate reserves.

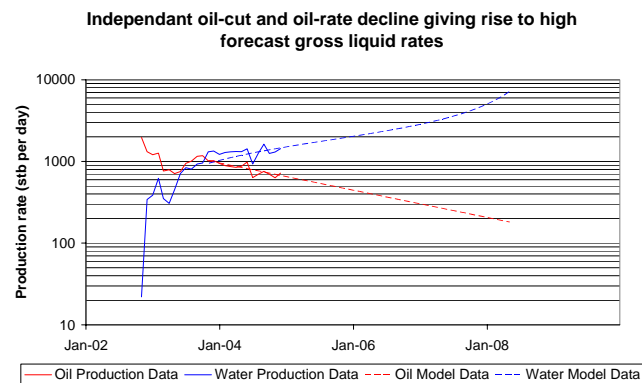


Figure 10

This is an extreme case and does not occur for each well with high BSW. But it should be noticed as it has an impact on water production and therefore facilities constraints.

Conclusions

At Addax Petroleum, operators of 60,000 stb/day of oil production in Nigeria, the method of production forecasting for existing wells is based on decline curve analysis. In order to have reasonable forecasts of not only oil but also water production, Addax uses, as standard, the method of specifying independent oil-rate and oil-cut declines, as implemented in the Serafim “FUTURE” production forecasting application.

For wells showing non-exponential decline, Addax uses generally hyperbolic decline curves, but also makes use of an alternative, more general form of decline curve, called the “C”-curve, which was first developed in order to construct a system model of the heavy-oil Alba field.

Both methods have been at least partially validated by the widespread use of them made by Addax. They may also prove useful to other oil companies.

Acknowledgements

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Nomenclature

(Note – there are some differences between nomenclature in the main body of the report and in the appendices).

a – decline constant used in C-curve; *a*=1 gives exponential decline

b – Arps *b* constant; C-curve *b* constant; when *a* = 0, if the relationship between the hyperbolic and C-curve *b* is $b_c = b_H / (1 - b_H)$, then the two curves are identical (see Appendix B)

N_p or *Q* – Cumulative oil production

q – oil production rate

R or *UR* – Technical ultimate recovery (to the limit of the decline curve)

r – fraction of ultimate recovery remaining = $(UR - N_p) / UR$

UR or *R* – Technical ultimate recovery (to the limit of the decline curve)

x – a measure of field aging, e.g. time, dimensionless time, cumulative liquid production

α, β - constants

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Appendix A

The justification for exponential decline is usually quoted as the empirical observation that

$$\frac{dq_{oil}}{dt} / q_{oil} = -a \quad (A-1)$$

This is then extended to hyperbolic decline in the form

$$\frac{dq_{oil}}{dt} / q_{oil} = -a \cdot q_{oil}^b \quad (A-2)$$

However, examination of the physics of simple systems that exhibit exponential decline (such as the decay of radioactive particles, or production of a gas field under pure depletion) suggests that the fundamental driver is that the decay/production rate is proportional to the remaining population/reserves. In our case, this would be

$$\frac{dQ_{oil}}{dt} / (R - Q_{oil}) = a \quad (A-3)$$

where

Q_{oil} = cumulative oil production

R = ultimate recovery

The "C-curve" method is to extend this relationship to the more general form

$$\frac{dQ_{oil}}{dt} / (R - Q_{oil}) = (a + \beta(R - Q_{oil})^b) \quad (A-4)$$

or, in a dimensionless form

$$\frac{dr}{dx} / r = -(a + \beta r^b) \quad (A-5)$$

where $r = 1 - Q_{oil} / R$

and x is a measure of field aging, such as Q_{liquid} / R

The equivalent relationship in hyperbolic decline, with Arps b is (see Appendix B)

$$\frac{dr}{dx} / r = -\beta r^{b/(1-b)} \quad (A-6)$$

The key idea behind this approach is that it is the $dr/dx - r$ relationship that matters for creating life-of-well or life-of-field production profiles. The exact form of the relationship

chosen does not matter much, providing it is sufficiently general to fit the shape of decline as observed in reality or as predicted in Eclipse. The C-curve relationship was chosen so as to be easily solvable to yield formulae that can be easily used and manipulated.

Solving for cumulative oil

Starting from the initial equation

$$\frac{dr}{dx} / r = -(a + \beta r^b) \quad (A-7)$$

the variables can be split as follows

$$\frac{1}{r(a + \beta r^b)} dr = -dx \quad (A-8)$$

Integrating both sides gives

$$\frac{1}{ab} \ln \left(\frac{r^b}{a + \beta r^b} \right) = -(x + \alpha) \quad (A-9)$$

where α is a constant

[Proof of integration of left hand side –

$$\begin{aligned} \frac{d}{dr} \left(\frac{1}{ab} \ln \left(\frac{r^b}{a + \beta r^b} \right) \right) &= \frac{d}{dr} \left(\frac{1}{ab} \ln \left(\frac{1}{ar^{-b} + \beta} \right) \right) \\ &= \frac{1}{ab} (ar^{-b} + \beta) \cdot \frac{-1}{(ar^{-b} + \beta)^2} \cdot (-abr^{-b-1}) = \frac{1}{r(a + \beta r^b)} \end{aligned} \quad (A-10)$$

Solving equation (A-9) for r gives

$$r = \sqrt[b]{\frac{a}{e^{ab(x+\alpha)} - \beta}} \quad (A-11)$$

The usual boundary conditions include

- production starts with dry oil i.e. $dr/dx = 1$ when $r = 1$
- At the start of production (i.e. when $r = 1$) $x = 0$

These boundary conditions allow us to express the α and β in terms of other variables, as follows

Condition (a) implies (from equation (A-7))

$$\beta = 1 - a \quad (\text{A-12})$$

Applying condition (b) to equation (A-9) gives

$$\frac{1}{ab} \ln \left(\frac{1}{a + (1-a)l^b} \right) = -(0 + \alpha) \quad (\text{A-13})$$

i.e.

$$\alpha = 0 \quad (\text{A-14})$$

Applying these values of α and β to equation (C) gives

$$r = \sqrt[b]{\frac{a}{e^{abx} + a - 1}} \quad (\text{A-15})$$

Changing from r and x to R , Q_{liquid} and Q_{oil} gives

$$Q_{\text{oil}} = R \cdot \left[1 - \sqrt[b]{\frac{a}{a-1 + e^{\left(\frac{a \cdot b \cdot Q_{\text{liquid}}}{R}\right)}}} \right] \quad (\text{A-16})$$

Appendix B

In this appendix we want to show that harmonic and C-curve decline are equivalent for appropriately chosen exponents.

Starting from the C-curve equation (A-4):

$$\frac{dQ}{dt} = \alpha (UR - Q)^{(b_c+1)} \quad (\text{B-1})$$

we can differentiate both sides with respect to Q so that

$$\frac{d}{dQ} \frac{dQ}{dt} = \alpha' (UR - Q)^{b_c} \quad (\text{B-2})$$

Using eq. (B-1) we can write the right hand side of the above equation as

$$\alpha' (UR - Q)^{b_c} = \alpha'' \left(\frac{dQ}{dt} \right)^{\frac{b_c}{b_c+1}} = \alpha'' q^{\frac{b_c}{b_c+1}} \quad (\text{B-3})$$

The left hand side of equation (B-2) can be rewritten as

$$\frac{d}{dQ} \frac{dQ}{dt} = \frac{dt}{dQ} \frac{d}{dt} \left(\frac{dQ}{dt} \right) = \frac{1}{q} \frac{dq}{dt} \quad (\text{B-4})$$

Substituting equations (B-3), (B-4) into (B-2) we have

$$\frac{\frac{dq}{dt}}{q} = \alpha'' q^{\frac{b_c}{b_c+1}} \quad (\text{B-5})$$

By comparing the above equation with (A-2) we can equate the exponents of q .

$$b_h = \frac{b_c}{b_c + 1} \Leftrightarrow b_c = \frac{b_h}{1 - b_h} \quad (\text{B-6})$$

Therefore one can conclude that harmonic decline with exponent b_h is equivalent to C-curve decline with exponent b_c for $a=0$.

Appendix C

In some cases of decline, an extra condition on the second derivative can be used. More specifically when the rate of production remains approximately constant for some duration of time we can use:

$$\frac{d}{dr} \left(\frac{dr}{dx} \right) = 0 \text{ for } r = 1 \quad (\text{C-1})$$

Starting with equation (A-8):

$$\frac{d}{dr} \left(\frac{dr}{dx} \right) = \frac{d}{dr} (-ar - \beta r^{b+1}) = -a - \beta(b+1)r^b \quad (\text{C-2})$$

and using equations (A-12), (C-1)

$$\begin{aligned} \frac{d}{dr} \left(\frac{dr}{dx} \right) &= -a - (1-a)(b+1) = 0 \Leftrightarrow \\ a &= 1 + \frac{1}{b} \end{aligned} \quad (\text{C-3})$$

This allows us to eliminate one of the two parameters from the C-curve equation.